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Generalized list matrix partition problems on chordal graphs, parameterized by leafage

Flavia Bonomo

Graph k-coloring and k-clique cover are examples of partition problems in graphs, in the first case into k independent sets, in the second case into k cliques. Moreover, maximum clique and maximum independent set are examples of partition problems into two sets, one arbitrary and the other one required to be a clique (resp. independent set), with the addition of a linear objective function to maximize. These are examples of matrix partition problems. For each symmetric matrix M over $\{0, 1, *\}$, the M-partition problem seeks a partition of the input graph into independent sets, cliques, or arbitrary sets, with certain pairs of sets being required to have no edges joining them, or to have all edges joining them, as encoded in the matrix. Moreover, the vertices of the input graph can be equipped with lists, restricting the parts to which a vertex can be placed. Even if the first four problems (k-coloring, k-clique cover, maximum clique and maximum independent set) are polynomially solvable on chordal graphs, Feder, Hell, Klein, Nogueira and Protti in 2005 proved that there are M-partition problems (without lists or objective functions) that remain NP-complete for chordal graphs. In this talk, making use of a graph width parameter called "thinness", we will show that all list matrix partition problems, with linear objective functions and further set cardinality restrictions, are XP on chordal graphs, parameterized by the leafage of the chordal graph. (The leafage of a chordal graph is the minimum number of leaves in a tree such that the graph can be realized as an intersection graph of subtrees of that tree.)

Twin-width, graph classes and a bit of logic

Eunjung Kim

Twin-width is a graph parameter introduced in 2020 by Bonnet, Kim, Thomassé, Watrigant, which quickly gained much traction since then. The contraction of two vertices u, v, in a graph G is an operation that identifies u and v into a new vertex z, and updates the adjacency relations between z and x of $V - \{u, v\}$ into (black) edge, non-edge and red edge: each reflects the state that $\{u, v\}$ and $\{x\}$ are homogeneously adjacent/homogeneously non-adjacent/not homogeneous. Twin-width of a graph G is the minimum number that upper bounds the red degree of all vertices along the contraction sequence from G to a single-vertex graph.

Many well-studied graph classes like bounded tree-width and rank-width graphs, unit interval graphs, strict hereditary classes of permutations, minor-closed graph classes are known to have bounded twin-width, thus the list of bounded twin-width classes include both spare and dense classes. As such, the notion twin-width provides a new perspective to explain why so many computational problems are fixed-parameter tractable on graph classes seemingly so different. Moreover, natural variants of twin-width give an alternative characterization of graph class of bounded rank-width, tree-width and so on.

For some graph classes such as ordered graphs, interval and permutation graphs, and tournaments, there is a strong indication that twin-width is the "right" measure to indicate the tameness of a graph class. That is, when a graph class C consists of ordered / interval / permutation graphs, the boundedness versus unboundedness of twin-width of C dictates the behaviors of the class, such as whether FO-model checking is tractable or not on C, whether the class of all graph can be "encoded" in the class C or not, etc. On the other hand, in general, twin-width is not a right notion for measuring the tameness of a class – the class of cubic graphs is tamed in the above sense, but the twin-width of a cubic graph can be arbitrarily large. We also review a recent effort to extend twin-width and capture a wider range of "tamed" graph classes.

Triangle-free graphs of large chromatic number

Nicolas Trotignon

Many constructions of triangle-free graphs of arbitrarily large chromatic number have been proposed since the early age of graph theory. We will survey them, focusing on the constructions of Zykov (1952), Mycielski (1955) and Burling (1965). We will present a new construction called twin-cut graphs, recently obtained by Édouard Bonnet, Romain Bourneuf, Julien Duron, Colin Geniet, Stéphan Thomassé and the speaker. They are structurally simple in several respects. For instance, they are all obtained from complete graphs on one or two vertices by repeatedly applying simple operations: disjoint union, gluing along a vertex, gluing along two vertices, and replicating a vertex into a pair of non-adjacent vertices. This answers a question by Chudnovsky, Scott, Penev and the speaker. Also, the hereditary class formed by their induced subgraphs can be recognized in polynomial time.

35 years of counting, sampling and mixing (WG Test of Time Award)

Mark Jerrum

The brief for this presentation indicated that it should be on the topic of a paper, with Alistair Sinclair, that appeared as an extended abstract in the proceedings of the 13th edition of this workshop, in 1987. At that time, the concepts listed in the title of the paper — approximate counting, uniform generation and rapidly mixing Markov chains — were only just beginning to be studied. (Nowadays, we would tend to use the term “sampling” rather than “generation”.) Nevertheless, the connections explored in the paper remain relevant today: that “conductance” (an expansion property of the state space of a Markov chain) implies mixing; that approximate counting and nearly uniform sampling are intimately linked, and that crude approximate sampling implies more precise approximate sampling. Although it could hardly be imagined at the time, these ideas were the seeds of a fruitful and coherent interdisciplinary research programme, driven by a host of talented theoretical computer scientists, probabilists and statistical physicists.

After sketching some of the main concepts, I shall take a problem-based approach, concentrating on approximately counting and sampling structures from graph theory. I shall aim to map the current frontier of research using some or all of following examples: matchings and perfect matchings, forests and connected spanning subgraphs, and vertex colourings. We have come a long way in 35 years, but there is more to explore.

Proportionally Fair Matching with Multiple Groups

Sayan Bandyapadhyay, Fedor V. Fomin, Tanmay Inamdar and Kirill Simonov

We study matching problems with the notion of proportional fairness. Proportional fairness is one of the most popular notions of group fairness where every group is represented up to an extent proportional to the final selection size. Matching with proportional fairness or more commonly, proportionally fair matching, was introduced in [Chierichetti et al., AISTATS, 2019]. In this problem, we are given a graph G whose edges are colored with colors from a set C. The task is for given $0 \leq \alpha \leq \beta \leq 1$, to find a maximum (α, β)-balanced matching M in G, that is a matching where for every color $c \in C$ the number of edges in M of color c is between $\alpha|M|$ and $\beta|M|$. Chierichetti et al. initiated the study of this problem with two colors and in the context of bipartite graphs only. However, in many practical applications, the number of colors — although often a small constant — is larger than two. In this work, we make the first step towards understanding the computational complexity of proportionally fair matching with more than two colors. We design exact and approximation algorithms achieving reasonable guarantees on the quality of the matching as well as on the time complexity, and our algorithms work in general graphs. Our algorithms are also supported by suitable hardness bounds.

Reconstructing Graphs from Connected Triples

Paul Bastide, Linda Cook, Jeff Erickson, Carla Groenland, Marc van Kreveld, Isja Mannens and Jordi Vermeulen

We introduce a new model of indeterminacy in graphs: instead of specifying all the edges of the graph, the input contains all triples of vertices that form a connected subgraph. In general, different (labelled) graphs may have the same set of connected triples, making unique reconstruction of the original graph from the triples impossible. We identify some families of graphs (including triangle-free graphs) for which all graphs have a different set of connected triples. We also give algorithms that reconstruct a graph from a set of triples, and for testing if this reconstruction is unique. Finally, we study a possible extension of the model in which the subsets of size k that induce a connected graph are given for larger (fixed) values of k.

Parameterized Complexity of Vertex Splitting to Pathwidth at most 1

Jakob Baumann, Matthias Pfretzschner and Ignaz Rutter

Motivated by the planarization of 2-layered straight-line drawings, we consider the problem of modifying a graph such that the resulting graph has pathwidth at most 1. The problem Pathwidth-One Vertex Explosion (POVE) asks whether such a graph can be obtained using at most k vertex explosions, where a vertex explosion replaces a vertex v by $\deg(v)$ degree-1 vertices, each incident to exactly one edge that was originally incident to v. For POVE, we give an FPT algorithm with running time $O(4^k \cdot m)$ and a quadratic kernel, thereby improving over the $O(k^6)$-kernel by Ahmed et al. [GD 22] in a more general setting. Similarly, a vertex split replaces a vertex v by two distinct vertices v_1 and v_2 and distributes the edges originally incident to v arbitrarily to v_1 and v_2. Analogously to POVE, we define the problem variant Pathwidth-One Vertex Splitting (POVS) that uses the split operation instead of vertex explosions. Here we obtain a linear kernel and an algorithm with running time $O((6k + 12)^k \cdot m)$. This answers an open question by Ahmed et al. [GD 22].

Odd Chromatic Number of Graph Classes

Rémy Belmonte, Ararat Harutyunyan, Noleen Köhler and Nikolaos Melissinos

A graph is called odd (respectively, even) if every vertex has odd (respectively even) degree. Gallai proved that every graph can be partitioned into two even induced subgraphs, or into an odd and an even induced subgraph. We refer to a partition into odd subgraph as an odd colouring of G. Scott [Graphs and Combinatorics, 2001] proved that a graph admits an odd colouring if and only if it has an even number of vertices. We say that a graph G is k-odd colourable if it can be partitioned into at most k odd induced subgraphs. We initiate the systematic study of odd colouring and odd chromatic number of graph classes. In particular, we consider for a number of classes whether they have bounded odd chromatic number. Our main results are that interval graphs, graphs of bounded modular-width and graphs of bounded maximum degree all have bounded odd chromatic number.

Deciding the Erdős-Pósa property in 3-connected digraphs

Julien Bensmail, Victor Campos, Ana Karolinna Maia, Nicolas Nisse and Ana Silva

A (di)graph H has the Erdős-Pósa (EP) property for the (butterfly) minors if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that, for any $k \in \mathbb{N}$ and any (di)graph G, either G contains at least k pairwise vertex-disjoint copies of H as (butterfly) minor, or there exists a subset T of at most $f(k)$ vertices such that H is not a (butterfly) minor of $G - T$. It is a well known result of Robertson and Seymour that an undirected graph has the Erdős-Pósa property if and only if it is planar. This result was transposed to digraphs by Amiri, Kawarabayashi, Kreutzer and Wollan, who proved that a strong digraph has the EP property for butterfly minors if, and only if, it is a butterfly minor of a cylindrical grid. Contrary to the undirected case where a graph is planar if and only if it is the minor of some grid, not all planar digraphs are butterfly minors of a cylindrical grid. In this work, we characterize the planar digraphs that have a butterfly model in a cylindrical grid. In particular, this leads to a linear-time algorithm that decides whether a weakly 3-connected strong digraph has the EP property.

New Width Parameters for Independent Set: One-sided-mim-width and Neighbor-depth

Benjamin Bergougnoux, Tuukka Korhonen and Igor Razgon

We study the tractability of the maximum independent set problem from the viewpoint of graph width parameters, with the goal of defining a width parameter that is as general as possible and allows to solve independent set in polynomial-time on graphs where the parameter is bounded. We introduce two new graph width parameters: one-sided maximum induced matching-width (o-mim-width) and neighbor-depth. O-mim-width is a graph parameter that is more general than the known parameters mim-width and tree-independence number, and we show that independent set and feedback vertex set can be solved in polynomial-time given a decomposition with bounded o-mim-width. O-mim-width is the first width parameter that gives a common generalization of chordal graphs and graphs of bounded clique-width in terms of tractability of these problems.

The parameter o-mim-width, as well as the related parameters mim-width and sim-width, have the limitation that no algorithms are known to compute bounded-width decompositions in polynomial-time. To partially resolve this limitation, we introduce the parameter neighbor-depth. We show that given a graph of neighbor-depth k, independent set can be solved in time $n^{O(k)}$ even without knowing a corresponding decomposition. We also show that neighbor-depth is bounded by a polylogarithmic function on the number of vertices on large classes of graphs, including graphs of bounded o-mim-width, and more generally graphs of bounded sim-width, giving a quasipolynomial-time algorithm for independent set on these graph classes. This resolves an open problem asked by Kang, Kwon, Strømme, and Telle [TCS 2017].

Nonplanar Graph Drawings with k Vertices per Face

Carla Binucci, Giuseppe Di Battista, Walter Didimo, Seok-Hee Hong, Michael Kaufmann, Giuseppe Liotta, Pat Morin and Alessandra Tappini

The study of nonplanar graph drawings with forbidden or desired crossing configurations has a long tradition in geometric graph theory, and received an increasing attention in the last two decades, under the name of beyond-planar graph drawing. In this context, we introduce a new hierarchy of graph families, called k^+-real face graphs. For any integer $k \geq 1$, a graph G is a k^+-real face graph if it admits a drawing Γ in the plane such that the boundary of each face (formed by vertices, crossings, and edges) contains at least k vertices of G. We give tight upper bounds on the maximum number of edges of k^+-real face graphs. In particular, we show that 1^+-real face and 2^+-real face graphs with n vertices have at most $5n - 10$ and $4n - 8$ edges, respectively. Also, if all vertices are constrained to be on the boundary of the external face, then 1^+-real face and 2^+-real face graphs have at most $3n - 6$ and $2.5n - 4$ edges, respectively. We also study relationships between k^+-real face graphs and beyond-planar graph families with hereditary property.

Computational Complexity of Covering Colored Mixed Multigraphs with Small Equivalence Classes in Degree Partition

Jan Bok, Jiří Fiala, Nikola Jedličková, Jan Kratochvíl and Michaela Seifrtová

The notion of graph covers (also referred to as locally bijective homomorphisms) plays an important role in topological graph theory and has found its computer science applications in models of local computation. For a fixed target graph H, the H-Cover problem asks if an input graph G allows a graph covering projection onto H. Despite the fact that the quest for characterizing the computational complexity of H-Cover had been started more than 30 years ago, only a handful of general results have been known so far.

In this paper, we present a complete characterization of the computational complexity of covering colored graphs for the case that every equivalence class in the degree partition of the target graph has at most two vertices. We prove this result in a very general form. Following the lines of current development of topological graph theory, we study graphs in the most relaxed sense of the definition - the graphs are mixed (they may have both directed and undirected edges), may have multiple edges, loops, and semi-edges. We show that a strong P/NP-co dichotomy holds true in the sense that for each such fixed target graph H, the H-Cover problem is either polynomial time solvable for arbitrary inputs, or NP-complete even for simple input graphs.

Cutting Barnette graphs perfectly is hard

Édouard Bonnet, Dibyayan Chakraborty and Julien Duron

A perfect matching cut is a perfect matching that is also a cutset, or equivalently a perfect matching containing an even number of edges on every cycle. The corresponding algorithmic problem, Perfect Matching Cut, is known to be NP-complete in subcubic bipartite graphs [Le and Telle, TCS '22] but its complexity was open in planar graphs and in cubic graphs. We settle both questions at once by showing that Perfect Matching Cut is NP-complete in 3-connected cubic bipartite planar graphs or Barnette graphs. Prior to our work, among problems whose input is solely an undirected graph, only Distance-2 4-Coloring was known NP-complete in Barnette graphs. Notably, Hamiltonian Cycle would only join this private club if Barnette's conjecture were refuted.

Metric dimension parameterized by treewidth in chordal graphs

Nicolas Bousquet, Quentin Deschamps and Aline Parreau

The metric dimension has been introduced independently by Harray, Melter and Slater in 1975 to identify vertices of a graph G using its distances to a subset of vertices of G. A resolving set X of a graph G is a subset of vertices such that, for every pair (u, v) of vertices of G, there is a vertex x in X such that the distance between x and u and the distance between x and v are distinct. The metric dimension of the graph is the minimum size of a resolving set. Computing the metric dimension of a graph is NP-hard even on split graphs and interval graphs. Bonnet and Purohit proved that the metric dimension problem is W[1]-hard parameterized by treewidth. Li and Pilipczuk strenghtened this result by showing that it is NP-hard for graphs of treewidth 24. In this article, we prove that that metric dimension is FPT parameterized by treewidth in chordal graphs.

Efficient Constructions for the Gyori-Lovasz Theorem on Almost Chordal Graphs

Katrin Casel, Tobias Friedrich, Davis Issac, Aikaterini Niklanovits and Ziena Zeif

In the 1970s, Gyori and Lovasz showed that for a k-connected n-vertex graph, a given set of terminal vertices $t_1, \ldots, t_k$ and natural numbers $n_1, \ldots, n_k$ satisfying $\sum_{i=1}^{k} n_i = n$, a connected vertex partition $S_1, \ldots, S_k$ satisfying $t_i \in S_i$ and $|S_i| = n_i$ exists. However, polynomial algorithms to actually compute such partitions are known so far only for $k \leq 4$. This motivates us to take a new approach and constrain this problem to particular graph classes instead of restricting the values of k. More precisely, we consider k-connected chordal graphs and a broader class of graphs related to them. For the first class, we give an algorithm with $O(n^2)$ running time that solves the problem exactly, and for the second, an algorithm with $O(n^4)$ running time that deviates on at most one vertex from the required vertex partition sizes.

Generating faster algorithms for d-Path Vertex Cover

Radovan Červený and Ondřej Suchý

Many algorithms which exactly solve hard problems require branching on more or less complex structures in order to do their job. Those who design such algorithms often find themselves doing a meticulous analysis of numerous different cases in order to identify these structures and design suitable branching rules, all done by hand. This process tends to be error prone and often the resulting algorithm may be difficult to implement in practice.

In this work, we aim to automate a part of this process and focus on simplicity of the resulting implementation. We showcase our approach on the following problem. For a constant d, the d-Path Vertex Cover problem (d-PVC) is as follows: Given an undirected graph and an integer k, find a subset of at most k vertices of the graph, such that their deletion results in a graph not containing a path on d vertices as a subgraph. We develop a fully automated framework to generate parameterized branching algorithms for the problem and obtain algorithms outperforming those previously known for $3 \leq d \leq 8$. E.g., we show that 5-PVC can be solved in $O(2.7^k n^{O(1)})$ time.

A new width parameter of graphs based on edge cuts: α-edge-crossing width

Yeonsu Chang, O-Joung Kwon and Myounghwan Lee

We introduce graph width parameters, called α-edge-crossing width and edge-crossing width. These are defined in terms of the number of edges crossing a bag of a tree-cut decomposition. They are motivated by edge-cut width, recently introduced by Brand et al. (WG 2022). We show that edge-crossing width is equivalent to the known parameter tree-partition-width. On the other hand, α-edge-crossing width is a new parameter tree-cut width and α-edge-crossing width are incomparable, and they both lie between tree-partition-width and edge-cut width.

We provide an algorithm that, for a given n-vertex graph G and integers k and α, in time $2^{O((\alpha+k)\log(\alpha+k))}n^2$ either outputs a tree-cut decomposition certifying that the α-edge-crossing width of G is at most $2\alpha^2 + 5k$ or confirms that the α-edge-crossing width of G is more than k. As applications, for every fixed α, we obtain FPT algorithms for the List Coloring and Precoloring Extension problems parameterized by α-edge-crossing width. They were known to be W[1]-hard parameterized by tree-partition-width, and FPT parameterized by edge-cut width, and we close the complexity gap between these two parameters.

Snakes and Ladders: a Treewidth Story

Steven Chaplick, Steven Kelk, Ruben Meuwese, Matúš Mihalák and Georgios Stamoulis

Let G be an undirected graph. We say that G contains a ladder of length k if the $(2 \times (k+1))$ grid graph is an induced subgraph of G that is only connected to the rest of G via its four cornerpoints. We prove that if all the ladders contained in G are reduced to length 4, the treewidth remains unchanged (and that this bound is tight). Our result indicates that, when computing the treewidth of a graph, long ladders can simply be reduced, and that minimal forbidden minors for bounded treewidth graphs cannot contain long ladders. Our result also settles an open problem from algorithmic phylogenetics: the common chain reduction rule, used to simplify the comparison of two evolutionary trees, is treewidth-preserving in the display graph of the two trees.

Parameterized Results on Acyclic Matchings with Implications for Related Problems

Juhi Chaudhary and Meirav Zehavi

A matching M in a graph G is an *acyclic matching* if the subgraph of G induced by the endpoints of the edges of M is a forest. Given a graph G and $\ell \in \mathbb{N}$, Acyclic Matching asks whether G has an acyclic matching of *size* (i.e., the number of edges) at least ℓ. In this paper, we first prove that assuming $\mathsf{W[1]} \nsubseteq \mathsf{FPT}$, there does not exist any FPT-approximation algorithm for Acyclic Matching that approximates it within a constant factor when parameterized by ℓ. Our reduction is general in the sense that it also asserts FPT-inapproximability for Induced Matching and Uniquely Restricted Matching as well. We also consider three below-guarantee parameters for Acyclic Matching, viz. $\frac{n}{2} - \ell$, $\mathsf{MM(G)} - \ell$, and $\mathsf{IS(G)} - \ell$, where n is the number of vertices in G, $\mathsf{MM(G)}$ is the *matching number* of G, and $\mathsf{IS(G)}$ is the *independence number* of G. We note that the result concerning the below-guarantee parameter $\frac{n}{2} - \ell$ is the most technical part of our paper. Also, we show that Acyclic Matching does not exhibit a polynomial kernel with respect to the vertex cover number (or vertex deletion distance to clique) plus the size of the matching, unless $\mathsf{NP} \subseteq \mathsf{coNP/poly}$.

$\mathcal{P}$-matchings Parameterized by Treewidth

Juhi Chaudhary and Meirav Zehavi

A *matching* is a subset of edges in a graph G that do not share an endpoint. A matching M is a $\mathcal{P}$*-matching* if the subgraph induced by the endpoints of the edges of M satisfies property $\mathcal{P}$. For example, if the property $\mathcal{P}$ is that of being a matching, being acyclic, or being disconnected, then we obtain an *induced matching*, an *acyclic matching*, and a *disconnected matching*, respectively. In this paper, we analyze the problems of the computation of these matchings from the viewpoint of Parameterized Complexity with respect to the parameter *treewidth*.

Algorithms and hardness for Metric Dimension on digraphs

Antoine Dailly, Florent Foucaud and Anni Hakanen

In the Metric Dimension problem, one asks for a minimum-size set R of vertices, such that for any pair of vertices of the graph, there is a vertex from R whose two distances to the vertices of the pair are distinct. This problem has mainly been studied on undirected graphs and has gained a lot of attention in the recent years. We focus on directed graphs, and show how to solve the problem in linear-time on digraphs whose underlying undirected graph (ignoring multiple edges) is a tree. This (nontrivially) extends a previous algorithm for oriented trees. We then extend the method to unicyclic digraphs (understood as those digraphs whose underlying undirected multigraph has a unique cycle). We also give a fixed-parameter-tractable algorithm for digraphs when parameterized by the directed modular-width, extending a known result for undirected graphs. Moreover, we show that Metric Dimension is NP-hard even on planar triangle-free acyclic digraphs of maximum degree 6.

Degreewidth: a New Parameter for Solving Problems on Tournaments

Tom Davot, Lucas Isenmann, Sanjukta Roy and Jocelyn Thiebaut

In the paper, we define a new parameter for tournaments called degreewidth which can be seen as a measure of how far is the tournament from being acyclic. The degreewidth of a tournament T denoted by $\Delta(T)$ is the minimum value k for which we can find an ordering $<v_1, \ldots, v_n>$ of the vertices of T such that every vertex is incident to at most k backward arcs (*i.e.* an arc (v_i, v_j) such that $j < i$). Thus, a tournament is acyclic if and only if its degreewidth is zero. Additionally, the class of sparse tournaments defined by Bessy *et al.* [ESA 2017] is exactly the class of tournaments with degreewidth one.

We study computational complexity of finding degreewidth.We show it is NP-hard and complement this result with a 3-approximation algorithm. We provide a cubic algorithm to decide if a tournament is sparse.

Finally, we study classical graph problems Dominating Set and Feedback Vertex Set parameterized by degreewidth. We show the former is fixed parameter tractable whereas the latter is NP-hard even on sparse tournaments. Additionally, we show polynomial time algorithm for Feedback Arc Set on sparse tournaments.

Approximating Bin Packing with Conflict Graphs via Maximization Techniques

Ilan Doron-Arad and Hadas Shachnai

We give a comprehensive study of bin packing with conflicts (BPC). The input is a set I of items, sizes $s : I \to [0,1]$, and a conflict graph $G = (I, E)$. The goal is to find a partition of I into a minimum number of independent sets, each of total size at most 1. Being a generalization of the notoriously hard graph coloring problem, BPC has been studied mostly on polynomially colorable conflict graphs. An intriguing open question is whether BPC on such graphs admits the same best known approximation guarantees as classic bin packing. We answer this question negatively, by showing that (in contrast to bin packing) there is no asymptotic polynomial-time approximation scheme (APTAS) for BPC already on seemingly easy graph classes, such as bipartite and split graphs. We complement this result with improved approximation guarantees for BPC on several prominent graph classes. Most notably, we derive an asymptotic 1.391-approximation for bipartite graphs, a 2.445-approximation for perfect graphs, and a $(1 + 2/e)$-approximation for split graphs. To this end, we introduce a generic framework relying on a novel interpretation of BPC allowing us to solve the problem via maximization techniques. Our framework may find use in tackling BPC on other graph classes arising in applications.

α_i-Metric Graphs: Radius, Diameter and all Eccentricities

Feodor Dragan and Guillaume Ducoffe

We extend known properties of chordal graphs and distance-hereditary graphs to much larger graph classes by using only a common metric property of these graphs. Specifically, a graph is called α_i-metric ($i \in N$) if it satisfies the following α_i-metric property for every vertices u, w, v and x: if a shortest path between u and w and a shortest path between x and v share the terminal edge vw, then $d(u,x) \geq d(u,v) + d(v,x) - i$. Roughly, gluing together any two shortest paths along a common terminal edge may not necessarily result in a shortest path but yields a "near-shortest" path with defect at most i. It is known that α_0-metric graphs are exactly ptolemaic graphs, and that chordal graphs and distance-hereditary graphs are α_i-metric for $i = 1$ and $i = 2$, respectively. We show that an additive $O(i)$-approximation of the radius, of the diameter, and in fact of all vertex eccentricities of an α_i-metric graph can be computed in total linear time. Our strongest results are obtained for α_1-metric graphs, for which we prove that a central vertex can be computed in subquadratic time, and even better in linear time for so-called (α_1, Δ)-metric graphs (a superclass of chordal graphs and of plane triangulations with inner vertices of degree at least 7). The latter answers to a question raised in (Dragan, *IPL*, 2020). Our algorithms follow from new results on centers and metric intervals of α_i-metric graphs. In particular, we prove that the diameter of the center is at most $3i + 2$ (at most 3, if $i = 1$). The latter partly answers to a question raised in (Yushmanov & Chepoi, *Mathematical Problems in Cybernetics*, 1991).

Bounds on Functionality and Symmetric Difference – Two Intriguing Graph Parameters

Pavel Dvořák, Lukáš Folwarczný, Michal Opler, Pavel Pudlák, Robert Šámal and Tung Anh Vu

[Alecu et al.: Graph functionality, JCTB2021] define functionality, a graph parameter that generalizes graph degeneracy. They research the relation of functionality to many other graph parameters (tree-width, clique-width, VC-dimension, etc.). Extending their research, we prove a logarithmic lower bound for functionality of random graph $G(n, p)$ for large range of p. Previously known graphs have functionality logarithmic in number of vertices. We show that for every graph G on n vertices we have $\text{fun}(G) \leq O(\sqrt{n \log n})$ and we give a nearly matching $\Omega(\sqrt{n})$-lower bound provided by projective planes.

Further, we study a related graph parameter *symmetric difference*, the minimum of $|N(u)\Delta N(v)|$ over all pairs of vertices of the "worst possible" induced subgraph. It was observed by Alecu et al. that $\text{fun}(G) \leq \text{sd}(G) + 1$ for every graph G. We compare fun and sd for the class INT of interval graphs and CA of circular-arc graphs. We let INT_n denote the n-vertex interval graphs, similarly for CA_n.

Alecu et al. ask, whether fun(INT) is bounded. Dallard et al. answer this positively in a recent preprint. On the other hand, we show that $\Omega(\sqrt[4]{n}) \leq \text{sd}(\text{INT}_n) \leq O(\sqrt[3]{n})$. For the related class CA we show that $\text{sd}(\text{CA}_n) = \Theta(\sqrt{n})$.

We propose a follow-up question: is fun(CA) bounded?

Maximum edge colouring problem on graphs that exclude a fixed minor

Zdeněk Dvořák and Abhiruk Lahiri

The maximum edge colouring problem considers the maximum colour assignment to edges of a graph under the condition that every vertex has at most a fixed number of distinct coloured edges incident on it. If that fixed number is q we call the colouring a maximum edge q-colouring. The problem models a non-overlapping frequency channel assignment question on wireless networks. The problem has also been studied from a purely combinatorial aspect in the graph theory literature. We study the question when the input graph is sparse. We show the problem remains NP-hard on 1-apex graphs. We also show that there exists PTAS for the problem on minor free graphs. The PTAS is based on a recently developed Baker game technique for proper minor-closed classes, thus avoiding the need to use any involved structural results. This further pushes the Baker game technique beyond the problems expressible in the first-order logic.

Cops and Robbers on Multi-layer Graphs

Jessica Enright, Kitty Meeks, William Pettersson and John Sylvester

We generalise the popular *cops and robbers* game to multi-layer graphs, where each cop and the robber are restricted to a single layer (or set of edges).

We show that initial intuition about the best way to allocate cops to layers is not always correct, and prove that the multi-layer cop number is neither bounded from above nor below by any function of the cop numbers of the individual layers. We determine that it is NP-hard to decide if k cops are sufficient to catch the robber, even if each layer is a tree plus some isolated vertices. However, we give a polynomial time algorithm to determine if k cops can win when the robber layer is a tree. Additionally, we investigate a question of worst-case division of a simple graph into layers: given a simple graph G, what is the maximum number of cops required to catch a robber

over all multi-layer graphs where each edge of G is in at least one layer and all layers are connected? For cliques, suitably dense random graphs, and graphs of bounded treewidth, we determine this parameter up to multiplicative constants. Lastly we consider a multi-layer variant of Meyniel's conjecture, and show the existence of an infinite family of graphs whose multi-layer cop number is bounded from below by a constant times $n/\log n$, where n is the number of vertices in the graph.

Parameterized Complexity of Broadcasting in Graphs

Fedor Fomin, Pierre Fraigniaud and Petr Golovach

The broadcast problem consists, given a graph G, and a source vertex s, to compute the minimum number of rounds required to disseminate an information from s to all vertices in the graph. It is assumed that, at each round, an informed vertex can transmit the information to at most one of its neighbors. The broadcast problem is NP-hard, even for simple graph classes such as cactus graphs. We show that the problem is FPT when parametrized by the size k of a feedback edge-set, or by the size k of a vertex-cover, or by $k = n - t$, where t is the input deadline for the broadcast protocol to complete.

Turan's Theorem Through Algorithmic Lens

Fedor Fomin, Petr Golovach, Danil Sagunov and Kirill Simonov

The fundamental theorem of Turán from Extremal Graph Theory determines the exact bound on the number of edges $t_r(n)$ in an n-vertex graph that does not contain a clique of size $r + 1$. We establish an interesting link between Extremal Graph Theory and Algorithms by providing a simple compression algorithm that in linear time reduces the problem of finding a clique of size ℓ in an n-vertex graph G with $m \geq t_r(n) - k$ edges, where $\ell \leq r + 1$, to the problem of finding a maximum clique in a graph on at most $5k$ vertices. This also gives us an algorithm deciding in time $2.49^k \cdot (n + m)$ whether G has a clique of size ℓ.

As a byproduct of the new compression algorithm, we give an algorithm that in time $2^{\mathcal{O}(td^2)} \cdot n^2$ decides whether a graph contains an independent set of size at least $n/(d + 1) + t$. Here d is the average vertex degree of the graph G. The multivariate complexity analysis based on ETH indicates that the asymptotical dependence on several parameters in the running times of our algorithms is tight.

On the Frank number and nowhere-zero flows on graphs

Jan Goedgebeur, Edita Máčajová and Jarne Renders

An edge e of a graph G is called *deletable* for some orientation o if the restriction of o to $G - e$ is a strong orientation. Inspired by an open problem of Frank, in 2021 Hörsch and Szigeti proposed a new parameter for 3-edge-connected graphs, called the Frank number, which refines k-edge-connectivity. The Frank number is defined as the minimum number of orientations of G for which every edge of G is deletable in at least one of them. They showed that every 3-edge-connected graph has Frank number at most 7 and that in case these graphs are also 3-edge-colourable graphs the parameter is at most 3. Here we strengthen the latter result by showing that such graphs have Frank number 2, which also confirms a conjecture by Barát and Blászik.

Furthermore, we prove two sufficient conditions for cubic graphs to have Frank number 2 and use them in an algorithm to computationally show that the Petersen graph is the only cyclically 4-edge-connected cubic graph up to 36 vertices having Frank number greater than 2.

On the minimum number of arcs in 4-dicritical oriented graphs

Frédéric Havet, Lucas Picasarri-Arrieta and Clément Rambaud

We prove that every 4-dicritical oriented graph on n vertices has at least $(\frac{10}{3} + \frac{1}{51})n - 1$ arcs.

Tight Algorithms for Connectivity Problems Parameterized by Modular-Treewidth (Best Student Paper)

Falko Hegerfeld and Stefan Kratsch

We study connectivity problems from a fine-grained parameterized perspective. Cygan et al. (TALG 2022) first obtained algorithms with single-exponential running time $c^{tw}n^{O(1)}$ for connectivity problems parameterized by treewidth (tw) by introducing the cut-and-count-technique, which reduces the connectivity problems to locally checkable counting problems. In addition, the obtained bases c were proven to be optimal assuming the Strong Exponential-Time Hypothesis (SETH). As only sparse graphs may admit small treewidth, these results are not applicable to graphs with dense structure. A well-known tool to capture dense structure is the modular decomposition, which recursively partitions the graph into modules whose members have the same neighborhood outside of the module. Contracting the modules, we obtain a quotient graph describing the adjacencies between modules. Measuring the treewidth of the quotient graph yields the parameter modular-treewidth, a natural intermediate step between treewidth and clique-width. While less general than clique-width, modular-treewidth has the advantage that it can be computed as easily as treewidth. We obtain the first tight running times for connectivity problems parameterized by modular-treewidth. For some problems the obtained bounds are the same as relative to treewidth, showing that we can deal with a greater generality in input structure at no cost in complexity. We obtain the following randomized algorithms for graphs of modular-treewidth k, given an appropriate decomposition:

* Steiner Tree can be solved in time $3^k n^{O(1)}$
* Connected Dominating Set can be solved in time $4^k n^{O(1)}$
* Connected Vertex Cover can be solved in time $5^k n^{O(1)}$
* Feedback Vertex Set can be solved in time $5^k n^{O(1)}$

The first two algorithms are tight due to known results and the last two algorithms are complemented by new tight lower bounds under SETH.

Cops and Robber - When Capturing is not Surrounding (Best Paper)

Paul Jungeblut, Samuel Schneider and Torsten Ueckerdt

We consider "surrounding" versions of the classic Cops and Robber game. The game is played on a connected graph in which two players, one controlling a number of cops and the other controlling a robber, take alternating turns. In a turn, each player may move each of their pieces: The robber always moves between adjacent vertices. Regarding the moves of the cops we distinguish four versions that differ in whether the cops are on the vertices or the edges of the graph and whether the robber may move on/through them. The goal of the cops is to surround the robber, i.e., occupying all neighbors (vertex version) or incident edges (edge version) of the robber's current vertex. In contrast, the robber tries to avoid being surrounded indefinitely. Given a graph, the so-called cop number denotes the minimum number of cops required to eventually surround the robber. We relate the different cop numbers of these versions and prove that none of them is bounded by a function of the classical cop number and the maximum degree of the graph, thereby refuting a conjecture by Crytser, Komarov and Mackey [Graphs and Combinatorics, 2020].

Complexity results for matching cut problems in graphs without long induced paths

Hoang-Oanh Le and Van Bang Le

In a graph, a (perfect) matching cut is an edge cut that is a (perfect) matching. Matching Cut (MC), respectively, Perfect Matching Cut (PMC), is the problem of deciding whether a given graph has a matching cut, respectively, a perfect matching cut. The Disconnected Perfect Matching problem (DPM) is to decide if a graph has a perfect matching that contains a matching cut. Solving an open problem recently posed in [Lucke, Paulusma, Ries (ISAAC 2022) & Feghali, Lucke, Paulusma, Ries (arXiv:2212.12317)], we show that PMC is NP-complete in graphs without induced 14-vertex path P_{14}. Our reduction also works simultaneously for MC and DPM, improving the previous hardness results of MC on P_{19}-free graphs and of DPM on P_{23}-free graphs to P_{14}-free graphs for both problems. Actually, we prove a slightly stronger result: within P_{14}-free graphs, it is hard to distinguish between

(i) those without matching cuts and those in which every matching cut is a perfect matching cut
(ii) those without perfect matching cuts and those in which every matching cut is a perfect matching cut
(iii) those without disconnected perfect matchings and those in which every matching cut is a perfect matching cut.

Moreover, assuming the Exponential Time Hypothesis, none of these problems can be solved in time $2^{o(n)}$ for n-vertex P_{14}-free input graphs. As a corollary from (i), computing a matching cut with a maximum number of edges is hard, even when restricted to P_{14}-free graphs. This answers a question asked in [Lucke, Paulusma amp Ries (arXiv:2207.07095)]. We also consider the problems in graphs without long induced cycles. It is known that MC is polynomially solvable in graphs without induced cycles of length at least 5 [Moshi (JGT 1989)]. We point out that the same holds for DPM.

Upper Clique Transversals in Graphs

Martin Milanič and Yushi Uno

A clique transversal in a graph is a set of vertices intersecting all maximal cliques. The problem of determining the minimum size of a clique transversal has received considerable attention in the literature. We initiate in this paper the study of the “upper” variant of this parameter, the upper clique transversal number, defined as the maximum size of a minimal clique transversal. We investigate this parameter from the algorithmic and complexity points of view, with a focus on various graph classes. We show that the corresponding decision problem is NP-complete in the classes of chordal graphs, chordal bipartite graphs, and line graphs of bipartite graphs, but solvable in linear time in the classes of split graphs and proper interval graphs.

Critical Relaxed Stable Matchings with Two-Sided Ties

Meghana Nasre, Prajakta Nimbhorkar and Keshav Ranjan

We consider the stable marriage problem in the presence of ties in preferences and critical agents. The input to our problem is a bipartite graph $G = (A \cup B, E)$ where A and B denote sets of agents which need to be matched. Each agent has a preference ordering over its neighbours possibly containing ties. In addition, a subset of agents in $A \cup B$ are marked as critical and the goal is to output a matching that matches as many critical agents as possible. Such matchings are called critical matchings in the literature. In our setting, we seek to compute a matching that is critical as well as optimal with respect to the preferences of the agents. Stability, which is a well-accepted notion of optimality in the presence of two-sided preferences, is generalized to weak-stability in the presence of ties. It is well known that in the presence of critical nodes, a matching that is critical as well as weakly stable may not exist. Popularity is another well-investigated notion of optimality for the two-sided preference list setting. However, in the presence of ties (even with no critical nodes), a popular matching need not exist. We, therefore, consider the notion of relaxed stability which was introduced and studied by Krishnaa et. al. (SAGT 2020). We show that a critical matching which is relaxed stable always exists in our setting although computing a maximum-sized relaxed stable matching turns out to be NP-hard. Our main contribution is a 3/2 approximation to the maximum-sized critical relaxed stable matching for the stable marriage problem with two-sided ties and critical nodes.

Graph Search Trees and Their Leaves

Robert Scheffler

Graph searches and their respective search trees are widely used in algorithmic graph theory. The problem whether a given spanning tree can be a graph search tree has been considered for different searches, graph classes and search tree paradigms. Similarly, the question whether a particular vertex can be visited last by some search has been studied extensively in recent years. We combine these two problems by considering the question whether a vertex can be a leaf of a graph search tree. We show that for particular search trees, including DFS trees, this problem is easy if we allow the leaf to be the first vertex of the search ordering. We contrast this result by showing that the problem becomes hard for many searches, including DFS and BFS, if we forbid the leaf to be the first vertex. Additionally, we present several structural and algorithmic results for search tree leaves of chordal graphs.