Riemannian Topology Meeting

9th and 10th November 2023, Fribourg (CH)

Location

All talks take place in lecture hall 2.52 of the building PER 08.

Date: 9th and 10th November

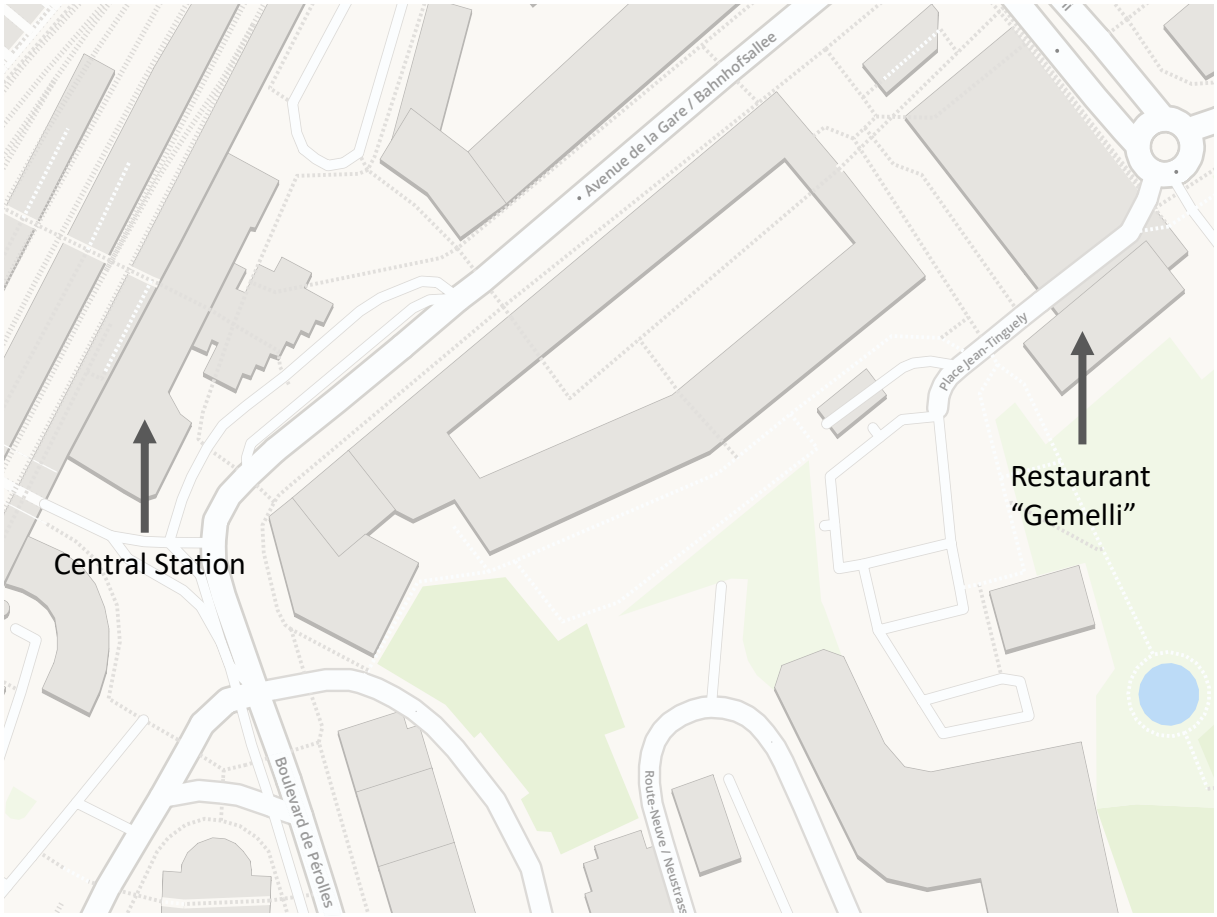
Venue: Lecture Hall 2.52, Building PER 08,
Chémin du Musée 3
1700 Fribourg
Switzerland

Conference Dinner

There will be a Conference Dinner on Thursday, the 9th of November.

Date: 9th of November, at 19h

Venue: Restaurant “Gemelli”
Grand-Places 10
1700 Fribourg
Switzerland



Program

Thursday, 9th November

10:15 – 11:05 (Rudolf Zeidler)

Nonnegative scalar curvature on manifolds with at least two ends

We study obstructions to the existence of complete positive scalar curvature (psc) metrics on manifolds with at least two ends in terms of the presence of certain hypersurfaces. Specifically, consider a compact two-sided connected incompressible hypersurface $Y \subset M$ which does not admit a metric of psc, where $6 \leq \dim M \leq 7$. In this case, if M and Y are either both totally non-spin or both almost spin, then M does not support a complete psc metric either. This result is motivated by questions of Gromov as well as Rosenberg-Stolz. Moreover, while the upper dimension restriction comes from the use of minimal hypersurface techniques, we show that the curious spin condition is in fact necessary for such a theorem to hold. Based on joint work with Simone Cecchini and Daniel Räde.

11:15 – 12:05 (Georg Frenck)

Surgery on fold maps

In this talk I will explore the notion of fold maps which are a natural generalization of Morse functions. Morse functions play a central role in the classification of manifolds and getting rid of their critical points is a crucial step in the proof of the h-cobordism theorem. I will describe a similar procedure for eliminating so called fold-singularities. This is similar in spirit but more flexible compared to the above-mentioned removal of critical points as it allows to perform surgery on the underlying manifold. If time permits I will also explain how this can be used to study fiber bundles and their characteristic classes.

Lunch break

13:30 – 14:20 *Minimal geodesics*
(Bernd Ammann)

A geodesic $c : \mathbb{R} \to M$ is called minimal if a lift to the universal covering globally minimizes distance. On the 2-dimensional torus with an arbitrary Riemannian metric there are uncountably many minimal geodesics. In dimension at least 3, there may be very few minimal geodesics. If M is closed, Bangert has shown that the number of geometrically distinct minimal geodesics is bounded below by the first Betti number b_1. In joint work with Clara Löh, we improve Bangert's lower bound and we show that this number is at least $(b_1)^2 + 2b_1$.

Coffee break

15:00 – 15:50 *Hyperbolic Coxeter groups of arbitrarily high vcd that virtually*
(Gangotryi Sorcar) *fiber over $\mathbb{Z}$.*

In this joint work with Jean-François Lafont, Barry Minemyer, Matthew Stover, and Joseph Wells, we provide an iterative procedure that produces infinitely many isomorphism classes of hyperbolic Coxeter groups in arbitrarily high virtual cohomological dimension $n \geq 3$ that virtually fiber over $\mathbb{Z}$.

16:00 – 16:50 *Extension of Möbius boundary homeomorphisms*
(Urs Lang)

In this talk, I will review recent results of K. Biswas. It is an open problem whether every Möbius homeomorphism between the visual boundaries of two Hadamard manifolds of curvature at most -1 extends to an isometry between them. A positive answer would resolve the long-standing marked length spectrum rigidity conjecture of Burns-Katok for closed negatively curved manifolds. Biswas' results yield an isometry between certain functorial thickenings of the manifolds, which lie within uniformly bounded distance and can be identified with their injective hulls.

Friday, 10th November

10:15 – 11:05 *Positive intermediate Ricci curvature on connected sums*
(Philipp Reiser)

Positive intermediate Ricci curvature is a family of interpolating curvature conditions between positive sectional and positive Ricci curvature. In this talk I will consider the problem of performing connected sums in the context of positive intermediate Ricci curvature. By generalizing work of Perelman and Burdick for positive Ricci curvature, I will show that such connected sums are possible, provided the manifolds involved possess "k-core metrics", which are metrics of positive intermediate Ricci curvature that contain a round hemisphere. As an example I will construct k-core metrics on complex and quaternionic projective spaces and the Cayley plane, and show that connected sums of linear sphere bundles over manifolds with k-core metrics admit a metric of positive intermediate Ricci curvature. This is joint work with David Wraith.

11:15 – 12:05 *Reading Topological ellipticity of G-manifolds from their quotients*
(Marco Radeschi)

Rational ellipticity is a very strong condition on a topological space, which in particular forces it to have "simple topology". Given its conjectured relation to manifolds with non-negative sectional curvature, a number of previous works has focused on finding geometric criteria that imply rational ellipticity. In this talk, I will describe a new criterion for a Riemannian G-manifold to be rationally elliptic, which generalizes most of the previously known ones. As an application, we will prove that non-negatively curved manifolds with an infinitesimally polar cohomogeneity 3 action must be rationally elliptic. This is joint work with Elahe Khalili Samani.

Lunch break

13:30 – 14:20 (Peter Quast)

Submanifolds of Clifford type, extrinsically symmetric spaces and a theorem of Loos.

We explain that compact extrinsically symmetric spaces of a Euclidean space are characterized by the property that each of its geodesics is contained in a totally geodesic submanifold which is extrinsically a Clifford torus. This yields a new approach to the following result, which is originally due to Ottmar Loos: Every compact symmetric space with rectangular unit lattice admits an extrinsically symmetric embedding into a Euclidean space. This is joint work with Ernst Heintze and Jost-Hinrich Eschenburg.

Coffee break

15:00 – 15:50 (Ivan Solonenko)

Homogeneous complex hypersurfaces in complex flag manifolds

A submanifold of a Riemannian manifold is called homogeneous if it is an orbit of an isometric Lie group action. For example, homogeneous real hypersurfaces are precisely nonsingular orbits of cohomogeneity-one actions. If the ambient manifold is Kähler, one can also consider the notion of homogeneous complex hypersurface (HCH). Smyth and Nomizu proved that in simply connected complex space forms, such hypersurfaces are very scarce: up to congruence, the only examples are C^{n-1} in C^n, CH^{n-1} in CH^n, CP^{n-1} in CP^n, as well as the smooth quadric Q^{n-1} in CP^n. The scarcity of HCHs persists in a much larger class of spaces, namely in complex flag manifolds with $b_2 = 1$; this class includes all irreducible compact Hermitian symmetric spaces. Using techniques from algebraic geometry, Konno was able to show that the only HCHs in complex flag manifolds with $b_2 = 1$ are the familiar CP^{n-1} and Q^{n-1} in CP^n, the subquadric Q^{n-1} in Q^n, as well as two more examples in the Grassmannian $Gr(2, C^{2n})$ and the exceptional symmetric space $E_6/Spin(10)U(1)$.
I will talk about some remarkable geometric properties of HCHs in Konno's classification. It turns out that the equidistant tubes around an HCH are contact hypersurfaces, whereas its focal set is always a projective space over a normed real division algebra. What is more, the ambient space can be regarded as the complexification of that focal set. Time permitting, I will mention some unresolved problems.

16:00 – 16:50 (Daniele Semola)

Ricci Curvature, Fundamental Groups, and the Milnor Conjecture

It was conjectured by John Milnor in 1968 that the fundamental group of a complete Riemannian manifold with nonnegative Ricci Curvature is finitely generated. I will present joint work with Elia Bruè and Aaron Naber where we construct a complete 7-dimensional Riemannian manifold with nonnegative Ricci Curvature and infinitely generated fundamental group, thus providing a counterexample to the Milnor conjecture.